Meeting the Mathematics Needs of 21st-Century Students with Math Techbook™

Overview

The world around us is interwoven with mathematics. Science has revealed that the natural world follows mathematical patterns previously unsuspected in such phenomena as migratory habits, organism growth, storm formation, and the structure of the universe. Engineers use mathematics to design, build, and test devices that were only the dreams of yesterday, but now the commonplace realities of today. However, many students do not recognize or appreciate the mathematics found in their everyday experiences, in exciting news events, or in the latest new device or trend. Too often, mathematics is something learned only to get to the next level of the subject or to satisfy a graduation requirement.

Discovery Education is developing an online, basal resource, Discovery Education Math Techbook, which engages students in connecting the real world to the mathematics necessary to achieve college and career readiness. Based on respected research and reports from the last 30 years about what works for students in understanding mathematical concepts and becoming proficient in mathematics, Math Techbook takes advantage of the power that digital media possesses to engage students and provide them the tools to explore mathematics concepts in depth.

This paper relates the guiding principles of Discovery Education's philosophy about learning in math, explains each principle using supporting research and reports on accepted best practices, and demonstrates how Math Techbook is specifically designed to help students meet the expectations of the Common Core State Standards for Mathematics (CCSSM) and its vision for increased mathematics proficiency.

What works in mathematics education and how does Discovery Education Math Techbook™ help teachers to make it work?

1. Experiential Learning Is Essential

Math is not a spectator sport. Active learning, in which students are engaged in the development of their own understanding of mathematical concepts as well as becoming facile in solving problems involving mathematics, is an essential element of good math instruction. The CCSSM place a great emphasis on students becoming involved in building their own understanding of math concepts in its *Standards for Mathematical Practice*. A major thread woven through these eight standards is making sense of problems, using reasoning to construct viable arguments, and modeling real-world problems using mathematics.

A recent report on improving mathematical problem solving from the What Works Clearing House[1] provides this definition: "Problem solving involves reasoning and analysis, argument construction, and the development of innovative strategies." The report goes on to explain that mathematical problem solving is not an innate talent. Problem solving is learned through guided practice, by providing students with both routine and non-routine problems, and by guiding them through the process of developing their own strategies to solve those problems.

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[1] Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide* (NCEE 2012-4055). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/publications_reviews.aspx#pubsearch/. (p.6)

This report complements the *Publishers' Criteria for the Common Core State Standards for Mathematics*,[2] which emphasizes the importance of involving students in developing solutions and strategies and sharing those with other students in the form of mathematical arguments. The What Works Clearing House report adds the criticism that "Too often, traditional textbooks do not provide students rich experiences in problem solving . . . teachers often review the answers quickly without discussing what strategies students used to solve the problems or whether the solutions can be justified."[3]

In support of learning that requires students to be involved in building their own mathematical ideas, these two research-based publications emphasize the need for instruction that uses mathematical problem solving to explore new concepts and that provides scaffolding to enable students to explore mathematics and discover solution strategies. Such scaffolding includes opportunities to develop and try solutions without penalty (using digital tools and hands-on materials) and questioning that guides the student, rather than supplying the answer or providing a single algorithm along with a set of practice exercises.

Regarding experiential learning at the high school level, in its publication *Focus on High School Mathematics*,[4] the National Council of Teachers of Mathematics (NCTM) emphasizes a focus on reasoning and sense making in high school to prepare students for using mathematics in everyday life, in the workplace, and in the scientific and technical community. The paper cites the Programme for International Student Assessment (PISA) analysis from the 2006 implementation,[5] which strongly suggests that U.S. students are lacking in the "ability to apply mathematics to analyse, reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations." This trend did not improve in the 2012 PISA results.

Both the CCSSM and the *Publishers' Criteria* therefore emphasize the importance of building students' skills not only in analyzing and solving problems by reasoning, but also in being able to pose mathematical arguments and communicate them effectively to others. This type of experiential learning should result in students being able to recognize sound mathematical ideas or refute poor ones. Mathematically literate adults should be able to analyze a variety of situations, such as:

- Is a proposed personal investment based on sound mathematics?
- Is the number of males and females accepted to a university disproportionate to the number of applicants?
- How is speed related to a safe following distance when driving behind other cars on the highway?

How Math Techbook Supports Experiential Learning

Discovery Education Math Techbook utilizes a learning cycle—Discover, Practice, Apply (DPA). The first phase of each cycle, Discover, includes scenarios relevant to the concept being learned and a rich set of investigations that enable students to actively explore each scenario, develop a mathematical model, make and test conjectures, and solidify their own understanding. Students also find solutions for problems that appear within the Discover section using what they've previously learned or discovered. Within each concept, students encounter multiple problems of increasing complexity based on the conceptual learning target specified by the CCSSM. The problems include a blend of online and hands-on investigations. Online investigations may contain interactives or scenarios that have been pre-loaded using one or more of the math tools,

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[2] *K–8 publishers' criteria for the common core state standards for mathematics.* Washington, DC: Authors. Retrieved from http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

[3] Woodward, J. ibid (p.6)

[4] The National Council of Teachers of Mathematics. (2009). Focus in high school mathematics: Reasoning and sense making. Position paper. Reston, VA.

[5] Programme for International Student Assessment (2007). *PISA 2006: Science competencies for tomorrow's world*. Paris: Organisation for Economic Co-operation and Development,304. http://www.pisa.oecd.org/dataoecd/30/17/39703267.pdf

which include a Graphing Calculator, Interactive Whiteboard, and Geometry Tool. Math tools are available at any time for students to use to explore concepts or solve problems.

For example, in working toward an understanding of the concept of the Pythagorean theorem, students first explore problems with right triangles that reinforce what they already know from previous years. Students are then challenged to identify relationships among the sides of a right triangle. Eventually, students are able to apply their understanding to finding the hypotenuse and explaining the relationships among the three sides in terms of squares of the values.

Each concept in Math Techbook contains a model lesson that provides an instructional pathway for teachers to take students through this exploration step by step. Digital and hands-on resources are used to enable guided exploration. The model lesson emphasizes that different students will apply different strategies and develop different solutions based on their prior understanding. Students are asked to share their solutions and explain their reasoning, developing their ability to propose mathematical arguments. The unique features of a digital environment make it possible not only for student exploration, but also for teachers to monitor student responses, share student examples with the class for discussion, and track student progress in their thinking. More will be shared about this capability in the section on monitoring and assessment.

2. Procedural Fluency Is Essential

To achieve proficiency in problem solving, both the National Mathematics Advisory Panel (NMAP) report and the *Publishers' Criteria* emphasize the importance of what is called procedural skill and fluency. As students build their understanding of mathematics, they move from simpler to more complex problems. In the early grades, the CCSSM references fluency primarily in terms of arithmetic operations. It is critical that students be able to quickly and readily solve arithmetic problems in order to use them while focusing on more complex mathematics, such as slope equations. However, fluency is not restricted to the operations domain. Students in the middle grades, for example, must be fluent enough with linear equations that they can build on that understanding as they move into exploring quadratic, exponential, and other nonlinear relationships.

The High School Publishers' Criteria specifically references the need for fluency: "…fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8)."[6] The Partnership for Assessment of Readiness for College and Careers (PARCC) also makes specific recommendations for fluency in the categories of algebra and functions.[7]
Instruction that promotes fluency and proficiency must foster comprehension and maintain it as students practice problems and exercises. Once students comprehend a mathematical idea or relationship and gain understanding of how they can solve a related problem, they need to then be given opportunity to become proficient at solving similar problems that use the same or a similar solution method. It is important to note that determining which solution method should be used is an important mathematical practice and is one example of how conceptualization and fluency are interwoven. It is also incumbent on the problem designers to ensure that practice problems are not simply clones of the problems used by students in gaining comprehension, but that practice problems vary enough to require that a true understanding of the mathematics is demonstrated.

How Math Techbook Supports Fluency

The Practice phase of Discovery Education Math Techbook's Discover-Practice-Apply (DPA) learning cycle is designed to develop procedural fluency and reinforce concepts. After

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[6] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2013). *High school publishers' criteria for the common core state standards for mathematics*. Washington, DC: p. 9.
[7] Partnership for Assessment of Readiness for College and Careeers. *Fluency recommendations (algebra II).* Retrieved from: http://www.parcconline.org/mcf/mathematics/fluency-recommendations-2.

demonstrating understanding within Discover, students move to Practice, which offers two types of support. The first is a guided pathway called Coach, in which students practice solving exercises similar to those that were solved in Discover. Exercises in Coach are purposefully varied so that students must apply their understanding of the concept to solve them, not just the algorithm or problem solution method they developed. In Coach, students' responses are monitored, and the system provides feedback based on the most common student errors and misconceptions. Students and teachers have access to a dashboard that tracks their attempts. After three attempts, students are provided with a text-based solution. Students can move freely through the items without needing to solve them in sequence.

The second level of support within Practice, called Play, consists of exercises through which students practice at their own pace. Students practice exercises based on the current concept. This area utilizes a gamification mode in which students track their progress and earn badges.

3. Application of Mathematics to the Real World Is Essential

The results of the most recent administration of the Programme for International Student Assessment (PISA) indicated that "students in the United States have particular weaknesses in performing mathematics tasks with higher cognitive demands, such as taking real-world situations, translating them into mathematical terms, and interpreting the model and results. An alignment study between the CCSSM and PISA suggests that a successful implementation of the Common Core Standards would yield significant performance gains."[8] This is supported by the Task Group Report of the National Mathematics Advisory Panel, which concluded that if mathematical ideas are taught using real-world contexts, students' performance on assessments involving similar problems improved.[9]

In *Focus in High School Mathematics – Reasoning and Sense Making*, the NCTM emphasizes the importance of making a connection between the mathematics that students are learning and its use in the real world. The term *mathematical modeling* is used to describe the process of connecting mathematics problem solving with a real-world situation. The goal is not only to use mathematics to help solve real-world problems, but also to recognize mathematics in the world around us. Most importantly, when posed in a context that engages students, mathematical modeling encourages students to acquire new mathematical understandings.[10]

The *Publishers' Criteria*, too, emphasizes that one of the three aspects of rigor in mathematics curricula is the application of mathematics to contextualized problems. Published math materials should "include a number of contextual problems that develop the mathematics of the course."[11] The *Publishers' Criteria* also cites building students' skill in developing mathematical models to solve problems.[12] The type of contextual problem, and how it is presented to students, however, is critically important.

Dan Meyer in a recent TED Talk contends that mathematics curricular materials have too often included formulaic word problems that give away the application of the mathematics and require the students to simply follow steps to a solution. He argues in favor of problems that are engaging, "worth solving," and posing them in such a way that students must determine the math

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[8] Organisation for Economic Cooperation and Development; Programme for International Student Assessment (PISA), *Results from PISA 2012; United States key findings.* p.1

[9] Gersten, R., Ferrini-Mundy, J., Benbow, C., Clements, D., Loveless, T., Williams, V., Arispe, I., and Banfield, M. (2008). *Report of the task force on instructional practices*. Washington, DC: National Mathematics Advisory Panel.

[10] The National Council of Teachers of Mathematics. (2009). Focus in high school mathematics: Reasoning and sense making. Position paper. Reston, VA.

[11] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2013). *High school publishers' criteria for the Common Core State Standards for Mathematics*. Washington, DC: p. 10.

[12] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2013 Ibid. p.10

application and the information needed to solve the problem.[13] Meyer states that engaging mathematical tasks should be brief and to the point, perplex the student to set up a state of curiosity, may involve pure mathematics that can be more interesting than attempting to force a problem in a real-world context, and should use photos or video instead of text to establish the context. Most importantly, he emphasizes that teachers should reveal information only when requested by students as they discuss and work through the problem in groups.[14]

The *Publishers' Criteria* argues that this type of problem setting encourages the development of mathematical practices, including constructing viable arguments and critiquing the arguments of others (MP.3). Student production and comprehension of mathematical arguments evolve through the grades from informal and concrete toward more formal and abstract.[15]

How Math Techbook Supports Application of Mathematics to the Real World

Although the last phase of Discovery Education Math Techbook's DPA learning cycle, Apply, focuses strongly on application of mathematics to the real world, engaging contextual problems are used throughout the DPA cycle and at every level of instruction in each concept. Units, which may contain multiple DPA learning cycles, begin with introductions that set up unanswered, real-world problems. Solutions are not provided to students. Instead, students are challenged by the end of the unit to develop and defend mathematical models that propose solutions using the understandings they gain during the unit learning progression.

In one example from Algebra II, the unit opener video describes scenarios of change: video popularity on social media, the growth of invasive species, and measuring the magnitude of an earthquake. During the unit, students learn how exponents and logarithms are used to mathematically model these types of changes to understand them better. By the end of the unit, having moved sequentially through problems in each concept of the unit that build their understanding, students can return to the original scenarios and explain how what they have learned can be used to build mathematical models of the situations.

Within each Discover phase, additional real-world context scenarios require students to develop solutions and explain reasoning. The unique ability of a digital resource is that student work is recorded for teacher review and class discussion. The way to understanding the larger problems is to pave the way with more manageable problems. In the Grade 7 study of proportional reasoning and scale, the unit opener shows how scale and proportion are important to art, mapping, and models, but leaves the problem questions open. In the concept 1 Discover for this unit, the pathway to solving the unit opener problem questions is paved with more manageable problems of exploring scale with animals, learning to write proportions using data about a new app, and applying the concept of proportion to a garden. The problems continue to build students' understanding and skill until they are ready to apply the understanding to a more complex problem.

By the final phase of the DPA learning cycle in each concept, Apply, students are given several contextual problems in which they must independently apply what they have learned during Discover and practiced through Practice to pose a solution and argument. Some problems are machine-scored; others use a digital rubric to allow for online scoring by the teacher. Students receive feedback on both types of items. The variety of problems in each concept progress from relatively straightforward application of the material to "messy" problems that add variables, require additional thought, expect students to conduct research and make assumptions, necessitate the creation of a mathematical model, and insist that students provide a well-articulated solution with justification.

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[13] Meyer, D. (May 2010). "Math class needs a makeover." Retrieved from http://www.ted.com/talks/dan_meyer_math_curriculum_makeover.html

[14] Meyer, D. (17 April 2012). Ten design principles for engaging math tasks. Retrieved from http://blog.mrmeyer.com/?p=12141,

[15] National Governors Association, ibid. p. 14.

The Apply problems involve students in modeling with mathematics. Teachers are encouraged to use them not only as assessment activities, but also as opportunities for classroom dialogue and discussion about effective mathematical modeling and problem solving.

4. A Focused and Coherent Curriculum Is Essential to Building Student Understanding and Achievement

When the results of the Third International Mathematics and Science Study (TIMSS) in 1996 reported that mathematics scores of students in the United States were far below the scores of their peers from other developed countries, a project was launched to determine how those countries differed from the United States regarding mathematics curricula and instruction. A 1997 report on a review of international curricula by the TIMSS concluded that, compared to curricula in countries with higher-achieving students, U.S. mathematics curricula attempted to teach too many topics in mathematics each year and were described as being “a mile wide and an inch deep.”[16] The result, the report concluded, was that teachers were rushing through the content and spending far too much time re-teaching what had been taught in previous grades. Subsequent international comparison studies have continued to search for what works. In 2009, the American Institutes for Research reviewed mathematics curricula from Hong Kong, Korea, and Singapore.[17] The ordering of the topics in these curricula was noteworthy to reviewers. Among the findings was that the curricula in these countries tended to present topics in logical order and presented competencies sequentially within a topic. The sequence and arrangement of topics was such that the study of one often reinforced material in other topics as well.

It is no surprise, then, that the NCTM and the National Mathematics Advisory Panel both have called for the development of a more focused and coherent mathematics curriculum.[18,19] In this sense, “focus” implies that each grade should concentrate on a few specific domains, spending significantly more instructional time on these to allow conceptualization to take place in the focus areas. Similarly, “coherence” means that topics are presented in a logical sequence, connecting to previous learning but also connecting ideas both within a domain and to other domains. For example, consider the reinforcing concepts of proportion, fractions, and similarity in geometric figures, each from a different domain.

The CCSSM target this idea of focus by strongly emphasizing specific areas for each grade level and course. They also model coherence vertically from grade to grade, referred to as progressions, and by clustering standards within a grade so that they are taught together. What the standards do not provide is a specific pathway through the content for each grade or specific recommendations for coherent connections between domains.

The *Publishers’ Criteria* provides some guidance in this area, emphasizing that in developing curricular materials, publishers should put the greatest emphasis on where the standards focus and link both within and across grades.[20] In addition, a companion essay to the *Publishers’ Criteria*, “The Structure is the Standards,”[21] emphasizes another form of coherency: not teaching a standard by itself, but instead organizing the curriculum so that clusters of related standards are taught together. The clustering of standards within CCSSM purposefully makes the connections among standards.

An illustrative example is represented by standards 8.EE.5 and 8.EE.6:

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[16] Schmidt, W.H., McKnight, C.C., and Raizen, S.A. (1997). *A Splintered vision: An investigation of U.S. science and mathematics education*. Dordrecht, The Netherlands: Kluwer.

[17] American Institutes for Research. (2009). *Informing grades 1-6 mathematics standards development: What can be learned from high performing Hong Kong, Korea, and Singapore*. Retrieved from http://www.air.org/files/MathStandards.pdf

[18] National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM. Retrieved from http://www.nctm.org/standards/content.aspx?id=3434

[19] National Mathematics Advisory Panel (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

[20] *K–8 publishers’ criteria for the Common Core State Standards for Mathematics.* Washington, DC: Authors. Retrieved from http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

[21] Daro, P., McCallum, B., and Zimba, J. (2012). “The structure is the standards.” Retrieved from http://commoncoretools.me/2012/02/16/the-structure-is-the-standards/

Cluster: Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Taken separately, the first standard, dealing with proportional relationships such as unit rate, would seem to have little in common with the similarity of triangles or the equation for slope. It is the cluster statement that ties them together and emphasizes that students should “understand the connections” so that they have a deeper understanding of proportional relationships.

At the high school level, the focus is specific to the college and career readiness standards, providing greater opportunity for students to comprehend the topics in depth. The *Publishers’ Criteria* cites feedback from surveys of post-secondary instructors, who prefer students to have “greater mastery of a smaller set of pre-requisites over shallow exposure to a wide array of topics.”[22] Thus, at the middle and high school levels, the publishing requirement is for greater focus on fewer skills to prepare students for postsecondary instruction, where greater depth of knowledge is rewarded.

A second emphasis in the companion essay to the *Publishers’ Criteria* is the importance of recognizing and following the progressions within domains. As with the curricula of the highest scoring nations on PISA, the CCSSM are organized in carefully crafted progressions from grade to grade, with each standard building on prior understandings. Knowing what students learned previously within a domain, as well as the related knowledge and skills, affords instruction that can remediate unfinished learning without the need to stop and reteach.

How Math Techbook Provides Focus and Coherence

The scope and sequence for each Discovery Education Math Techbook grade and course follows the directive to spend the majority of time on the focus topics for the course, narrowing the focus to what is required for the course in the standards, and making connections that follow the domain progressions and reach-across connections among domains. In the middle grades, the focus topics are addressed before the end of the third quarter, with the realization that testing typically occurs before the end of the year.

In developing Math Techbook’s scope and sequence of instruction, Discovery Education has also taken great care to maintain the continuity of clusters of standards. Units are organized to encompass clusters as a whole, using problem contexts that emphasize the connections among the standards in the cluster while following a step-by-step inquiry approach to allow students to explore each aspect of the standard and develop an understanding and facility in solving problems. Lesson objectives support the cluster as a whole, while targeting specific learning steps in the process of conceptualizing the main idea of the cluster.

In Math Techbook, the instruction within each concept includes connections to “reach-back” standards both in previous grades and from earlier in the year, as well as “reach-ahead” standards from later in the same course or in subsequent years. For example, in the Grade 8 concept, Represent Large and Small Numbers, the following progression is noted for the teacher:

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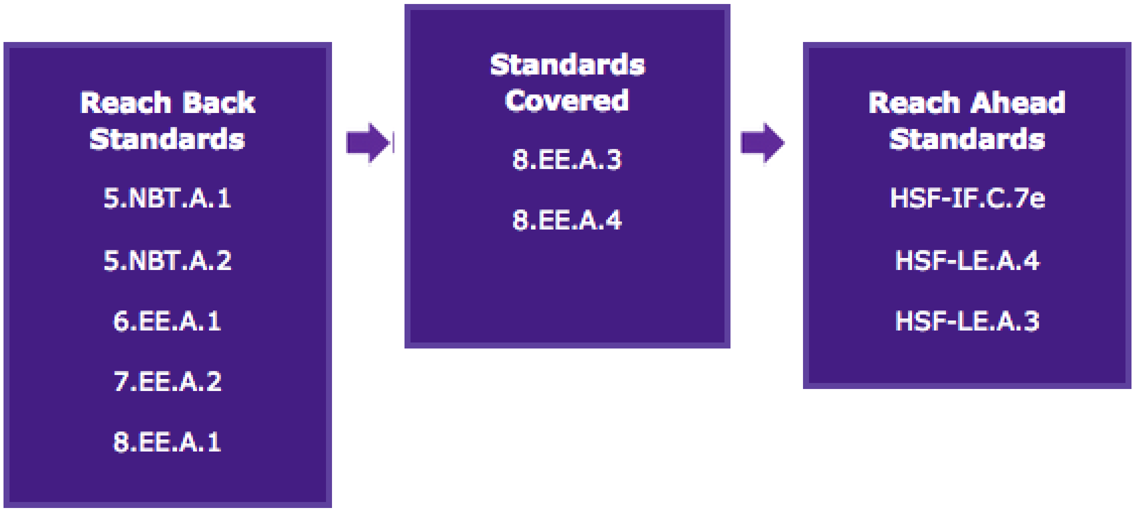
[22] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2013). *High school publishers’ criteria for the Common Core State Standards for Mathematics*. Washington, DC. http://www.corestandards.org/assets/Math_Publishers_Criteria_HS_Spring%202013_FINAL.pdf

Progressions and Common Core Standards



Below the diagram on every model lesson page are three explanatory paragraphs, one each to explain what should have been covered in previous years, what is covered in the current year, and what comes later. This exposition provides additional support to the teacher by expanding on the ideas given in the purple boxes.

The diagram above indicates to the teacher that in prior courses, students developed a conceptual understanding of place value relationships and patterns in the product when multiplying by powers of 10 (5.NBT.A.1, 5.NBT.A.2). They also developed a conceptual understanding of the properties of whole number and integer exponents, as well as learning how to generate equivalent numerical expressions (6.EE.A.1, 7.EE.A.2, 8.EE.A.1).

Through the investigations in this concept, students build on their knowledge of powers of 10 and writing multiplication expressions to estimate very large and very small quantities as a single digit multiplied by a power of 10 (8.EE.A.3). Then, they extend that knowledge to discover and practice writing numbers in scientific notation and selecting the correct units of measure (8.EE.A.4).

During subsequent instruction, students will compute with numbers written in scientific notation. Eventually, students will extend the properties of integer exponents to develop meaning for exponential and logarithmic functions using graphs and tables. They will use models and technology to explore and evaluate logarithms. (HSF-IF.C.7e, HSF-LE.A.4, HSF-LE.A.3)

5. The Standards for Mathematical Practice Should Be Woven into the Fabric of Student Learning Across All Topics in Math

The CCSSM Standards for Mathematical Practice evolved from two sources: the NCTM process standards[23] and the National Research Council's seminal publication, *Adding It Up.*[24] Each defined the importance of the practices as learning that should be implicitly taught and monitored. However, both publications cautioned that the practices are not skills to be mastered independently of math concepts. Rather, they are habits to be developed in the process of

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[23] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

[24] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Adding it up.*

conceptual development. The CCSSM include eight standards for mathematical practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

It should be noted that at the high school level, modeling with mathematics is so important that it is considered to be a conceptual category (the high school equivalent of a domain). Each standard that is appropriate for modeling with mathematics is identified within its category. The implied expectation is that real-world problems will be posed that require students to develop mathematical models and mathematical arguments that support the model around those standards.

Achieving the standards for mathematical practice requires students to do more than pencil-and-paper calculations.[25] Beyond memorizing procedures, students must develop a deeper understanding of mathematics, which involves students sometimes taking different, logical paths to reach the same solution. This change requires a move toward instructional practices that will give students greater opportunity to solve mathematics problems on their own, develop the skills for solving problems, and apply those skills to real-world problems.[26]

The *Rand Study Report: Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics*[27] cites an instructional example in which a student has argued that the number 6 could be even or odd. Although this example comes from the early grades, it is pertinent to every level of math instruction. In a traditional approach, the teacher could have simply explained why the number is even, but instead she allows the student to share his reasoning using a diagram and allows other students to debate the matter. When necessary, the teacher asks questions that eventually result in all of the students being able to explain and defend the accepted rule. This teacher was allowing the students to practice mathematics. The focus in this particular exercise is on MP.3, constructing viable arguments and critiquing the reasoning of others, but students also had to reason quantitatively (MP.2) and look for and make use of structure (MP.7). This activity is a prime example of solidifying the concept while at the same time developing math practices. The same approach can be used with students who pose any mathematical argument.

How Does Math Techbook Support the Standards for Mathematical Practice?

By using the type of conceptual approach to understanding that the Discover, Practice, Apply (DPA) cycle employs in Math Techbook, it is possible to set up problem solving so that students are able to employ the standards for mathematical practice across all concepts. Just as with the sample problem above, specific problems lend themselves to different standards, and teachers are encouraged to guide students to use the practices in developing arguments and gaining understanding.

The specific standards for mathematical practice emphasized in a concept within Math Techbook are identified in the model lesson for that concept, along with investigations that students conduct during that DPA cycle. Three standards for mathematical practice are identified as the focus standards for each unit, but they are selected such that adequate attention is given to all eight standards across the year of instruction.

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[25] Burns, M. (2012). Go figure: math and the common core. *Educational Leadership,* December 2012/January 2013: 42-46.

[26] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2013). *Frequently asked questions.* Retrieved from http://www.corestandards.org/resources/frequently-asked-questions

[27] Ball, D.L. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: RAND Corporation.

6. Ongoing Assessment and Data-Driven Decision Making by Teachers and Students Is Essential for Mathematical Growth and Development

In developing its national report, the National Mathematics Advisory Panel sought insight into what works in mathematics education. It charged the National Math Panel Task Group on Instruction to review the existing literature and research on assessment. Their report to the NMAP made clear that ongoing formative assessment is a significant factor in improving student learning in mathematics.[28]

The task group cautioned, however, that effective formative assessment has two requirements. First, teachers need guidance and professional development in using assessment to design and individualize instruction. Second, teachers are more willing to use ongoing assessment when it does not detract from other learning activities.

According to *Implementing Data-Informed Decision Making in Schools—Teacher Access, Supports and Use*[29] from the U.S. Department of Education, teachers who effectively use data-driven decision making require up-to-date information with a greater level of detail than formal assessments typically provide. Further, it concludes that technological formative assessment systems built into the instructional program can provide student information immediately and in greater detail.

A recognized leader in the field of assessment practices, Richard Stiggins makes the case that formative assessment and the instructional process should be inseparable:

> Effective formative assessment tells users if and when students are attaining the foundations of knowledge, the reasoning, the performance skills and the product development abilities that lead to mastery of essential content standards. However, the productive use of formative assessments depends on what teachers and students do with and about the test results (Stiggins, 2005).[30]

Formative assessment should be used to provide individualized feedback and information about what students need to improve performance and to provide instructional correctives, different from previous instruction, that will move the student toward successful attainment of learning targets.[31]

How Does Math Techbook Weave Ongoing Formative Assessment and Data-Driven Decision Making into Its Instructional Model?

Within the DPA cycle of Math Techbook is a formative assessment system woven into the instructional activities and practice tasks. As students respond to problems, their responses are collected and made available to the teacher through a student response panel. To ensure that teachers monitor deeper understanding, rather than simply looking for the correct responses, students are required to provide explanations, not just answers. Consequently, the learning activities mirror the types and levels of items and responses similar to those required on the PARCC and SBAC assessments currently being developed.

The two organizations developing national assessments for the CCSSM, the Partnership for

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[28] Gersten, R., Ferrini-Mundy, J., Benbow, C., Clements, D., Loveless, T., Williams, V., Arispe, I., and Banfield, M. (2008). *Report of the task force on instructional practices*. Washington, DC: National Mathematics Advisory Panel. Retrieved from www.ed.gov/about/bdscomm/list/mathpanel/index.html

[29] U.S. Department of Education, Office of Planning, Evaluation and Policy Development (2009). *Implementing data-informed decision making in schools—Teacher access, supports and use.* Retrieved from www.ed.gov/about/offices/list/opepd/ppss/reports.html

[30] Stiggins, R. (December 2005). From formative assessment to assessment for learning: A path to success in standards-based schools. *The Phi Delta Kappan, 87*, 324-328.

[31] Wiliam, D. (2010). An integrative summary of the research literature and implications for a new theory of formative assessment. In H. L. Andrade & G. J. Cizek (Eds.), *Handbook of formative assessment* (pp. 18-40). New York, NY: Taylor & Francis.

Assessment of Readiness for College and Careers (PARCC)[32] and Smarter Balanced Assessment Consortium (SBAC),[33] have designed assessments that provide a significant measure of conceptual attainment and problem-solving proficiency in mathematics. A key ingredient in the assessments are technology enhanced items, or TEIs. These items allow richer responses from students than traditional selected-response items. Almost all of the items are machine-scored, and individual student responses can be captured. Among the item types are sorting, placement (number line or other), constructed response (both free-form and machine-scored), classification, select and order, point placement, object transformation, partition and select, and others.

In Math Techbook, all three phases of the DPA cycle take advantage of the TEIs. They are not used exclusively on assessments but are woven throughout instruction and individual practice. As students solve problems in the Discover phase, they record their responses to questions and explain their thinking in text boxes. In some cases, students interact with digital or hands-on resources and then respond. For example, students might use a tool to plot data points on a graph and then explain their thinking about the pattern or submit an expression or equation that states the relationship mathematically.

Student responses are captured in a student response panel. This enables the teacher to monitor how students are responding as learning occurs. The teacher can react formatively to focus on individual students or to address the entire class. Because results on the dashboard can also be displayed anonymously, the teacher can reveal all responses to the entire class as a means of opening whole-class discussion about the problem and the multiple ways in which students are attempting to solve it. It is also important to note that students' responses persist within Techbook when they log off. Once they log in again, they can review the responses and even adjust them if desired.

In the Coach area of Practice, student responses are recorded and reported to the teacher on a separate dashboard, which also shows the number of attempts and whether or not a student viewed the video explanation of a problem. Items are aligned to the specific standard(s) they assess. Students have their own dashboard through which they can monitor their progress towards fluency and proficiency. In the Play section within Practice, a gamification system with rewards is used to further engage and encourage students, and scores are reported separately.

In the Apply phase, student responses to TEIs are recorded and machine-scored, and extended responses for richer problems can be evaluated by the teacher through an online scoring rubric. The combination of different types of assessment items focused on different parts of the learning cycle gives the teacher a full picture of student progress, with no loss of instructional time. Because students can log in from any location with Internet connectivity and on any device, the problems, exercises, responses, and any scores can be seen by parents as well.

7. Digital Resources Can Increase Engagement and Enable Active Involvement in Problem Solving

No one doubts that 21st-century students are consumers of digital technology content. But can that content be an aid to learning and achievement in mathematics?

Technology education authority, Cher Ping Lim, claims that digital content can be an aid to learning, if developed correctly. But he cautions, "State of-the-art Internet technologies [alone] do not ensure that learners are willing or know how to engage in the context of their learning and make sense of the information provided to construct their own knowledge."[34] He emphasizes the importance of contextualizing the learning activities so that the problems with which students are presented have meaning and allow them to utilize prior knowledge. Functionality is also important; it must take into account that teachers may need to guide students in acquiring

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[32] Partnership for Assessment of Readiness for College and Careers (PARCC). http://www.parcconline.org

[33] Smarter Balanced Assessment Consortium (SBAC). http://www.smarterbalanced.org

[34] Ling, P.C. (2004). Engaging learners in online learning environments. *Techtrends*, *48*,1. Association for Educational Communications and Technology.

learning strategies that enable them to fully benefit from an online learning component.

A major benefit of the use of an online digital resource is in providing ready access to tools for exploring as well as explaining mathematical ideas and arguments, devising problem solutions, and developing models. At the same time, digital resources can capture student responses, allowing both the teacher and student to better monitor their progress. In their paper, "Mathematical Knowledge and Practices Resulting from Access to Digital Technologies," Olive and Makar conclude the following: "The literature indicates that interactions among students, teachers, tasks, and technologies can bring about a shift in empowerment from teacher or external authority to the students as generators of mathematical knowledge and practices; and that feedback provided through the use of different technologies can contribute to students' learning."[35] They specifically find that technology, effectively employed, can enable students to "create meaning" and monitor their learning. Technology can control the complexity of a problem by adjusting the available inputs and outputs in interactive experiences. The authors caution that the technology is most effective when provided and directed by the teacher.

How Does Math Techbook Increase Student Engagement and Involvement in Problem Solving?

Math Techbook is based on an interactive discovery approach from the first screen. Each course page is interactive, allowing students to visualize the entire course and navigate to any part of it to explore more. Engaging video clips set up unit-long mathematics problems at the start of each unit.

The Discover phase of the DPA cycle in each concept presents real-world contexts for problems and intriguing math mysteries through a variety of media: text, interactives, video, and hands-on activities. As problems are presented, students are provided the tools to explore problem solutions and share them with one another, the teacher, and the class. At all levels, students create meaning through problem solving and monitor their own learning as the online resources collect and store their responses and ideas.

In the Practice phase of the cycle, students again monitor their own learning as they work through the Coach section. The Play area utilizes a gaming environment in which students build fluency as they receive awards from the system for achieving goals.

The Apply phase of the cycle poses additional engaging problems through the use of text and video and challenges students to apply their understanding of the concept in meaningful ways. Problems in Apply vary from direct application of the math concept to "messier" problems that challenge students to identify the pertinent information or to gather additional data from outside of the service in order to practice mathematical modeling.

An important distinction between Math Techbook and other online digital math resources is that Techbook is not designed as an alternative to the teacher, but rather serves as a primary resource for teaching and learning. The instructional model in Techbook promotes blended learning, i.e., learning in which online resources are used with face-to-face instruction and hands-on interactions.

8. Differentiation of Instruction Is Essential and Is Strongly Supported by Digital Resources

A major challenge for teachers is the variety of prior knowledge, skills, and strengths that their students bring to the classroom. There are three hurdles that must be overcome to ensure that students don't "fall through the cracks." The first is to understand what students understand. The second is to provide a means for meeting their individual learning needs. And the third is to

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[35] Olive, J., Makar, K., Hoyos, V., Kor, LK., Kosheleva, O., Sträßer, R. (2010) Mathematical knowledge and practices resulting from access to digital technologies**.** In C. Hoyles and J.B. Lagrange (Eds.). *Mathematics education and technology-rethinking the terrain* (133-177). US: Springer Science and Business Media.

monitor and adjust instruction based on feedback.

Digital resources provide a powerful tool for gathering data about students' understanding of mathematics and how this data can be used to inform instructional decisions on an individual basis. The *Publishers' Criteria* states that "Digital materials offer substantial promise for conveying mathematics in new and vivid ways and customizing learning."[36] It goes on to quote the CCSSM expectation that all students must have the opportunity to learn and meet the same high standards.

Teachers who recognize that students learn in different ways are familiar with the concept of multiple learning modalities (e.g., visual, auditory, textual, kinetic), where students have more strength in one modality than others or use them to support each other. Carol Ann Tomlinson[37] states that teachers embracing differentiated instruction recognize that students differ in important ways and engage students through different learning modalities. These teachers use varied rates of instruction and varying degrees of complexity. However, attending to this variety of learning styles can be a challenge.

In "Research into Visualization in Learning and Teaching Mathematics," Norma Presmeg cites the critical need to help all students learn to visualize mathematical concepts and make connections between visual and symbolic representations.[38] She notes the promise that technology has in providing imagery and connecting that imagery to symbolic representations for students who struggle with this essential skill. An example from Math Techbook is the graphing calculator, with which students can visualize changing values in equations while observing how those changes affect a trend or progression. For example, students who enter the function $y = ax + b$ are able to set sliders for a and b, and then they can adjust the sliders to determine the effect on the graph of the linear equation. Although this functionality is not new, there are very few curricula that have such robust tools included directly within the resource; most curricula require the use of hand-held calculators or external software. Interactive explorations, mathematical white boards, and even some TEI types afford this same opportunity for students to visually explore concepts.

Recent studies also report success by students who typically struggle in mathematics, particularly algebra and pre-algebra, when their teachers use an instructional methodology that promotes multiple-solution strategies.[39,40] Students benefit from sharing and comparing solution methods, and struggling students seem to also benefit, when guided through the process by a competent, trained teacher.

How Does Math Techbook Strongly Support Differentiation of Instruction?

Digital media in Techbook provides rich text, visual, auditory, and kinesthetic experiences that allow students to acquire the information they need in the format that best suits their learning style. Problems are presented through video, text, computer-based interactives, and hands-on experiences. Students are encouraged to use multiple modalities to solve problems, including the use of interactives developed specifically for a concept or any of the interactive math tools that can be used by students as they see fit. (Incidentally, allowing students to choose tools on their own leads toward MP.5, "use appropriate tools strategically.")

The highly evolved formative assessment system in Math Techbook enables teachers to easily

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[36] *K–8 publishers' criteria for the common core state standards for mathematics.* Washington, DC: Authors: 7. Retrieved from http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

[37] Tomlinson, C.A. (1999). *The differentiated classroom: Responding to the needs of all learners*. Association for Supervision and Curriculum Development.

[38] Presmeg, N. (2006). Research into visualization in learning and teaching mathematics. In Guttierrez, A. and Boero, P. (Eds.) *Handbook of research on the psychology of mathematics education: Past, present, and future*. Sense Publishers, Rotterdam.

[39] Star, J.R., and Rittle-Johnson, B. (2009). Making algebra work: Instructional strategies that deepen student understanding within and between representations. *ERS Spectrum, 27*, no 2: 11-18. Retrieved from http://nrs.harvard.edu/urn-3:HUL.InstRepos:4889486

[40] Lynch, K. and Star, J.R. (2014). Views of struggling students on instruction incorporating multiple strategies in algebra I: An exploratory study. *Journal of Research in Mathematics Education, 45(1),* 6-17.

monitor student responses and target specific needs. Teachers can identify student misconceptions, ask guiding questions, and present additional information in a variety of formats.

Even more promising are features that digital technology provides for special populations of students. Techbook provides a variety of support. Text to speech is pervasive throughout the service. Text size can be increased as needed. Glossary term definitions and explanations are provided in multiple modalities of text, animation, imagery, and video. Video is captioned. All online materials are 508 compliant, and interactivity is supported by multimodal output.

Finally, Discovery Education Math Techbook provides digital imagery that enables students to make connections between visual and symbolic representations. The instructional model promotes multiple solution strategies and facilitates teacher and student sharing and comparison. The Discover phase of the DPA cycle provides multiple opportunities for students to view and compare solution methods. Model lessons coach teachers in using the strategy, and professional development provides guidance and support in adopting the process.

How Are Highly Able Students Supported by Math Techbook?

The *Publishers' Criteria* is steadfast that, regardless of the specific pathway and sequence of instruction, students who are highly able in mathematics should be provided with enrichment problems and exercises that broaden and deepen their understanding of grade level mathematics concepts.[41] The *Publishers' Criteria* discourages the acceleration of such students to work with content from the next grade level. For example, students who demonstrate understanding of the Pythagorean theorem should be introduced to more complex patterns and applications using the theorem to identify Pythagorean triples.

Although schools are free to utilize later course content, in support of this requirement, each concept in Math Techbook contains an extension of the concept, found in the Discover phase. Here, students who "already get it" are challenged to extend their thinking into more complex problems around the standard being targeted. The activity is designed as a project that students work on to extend their understanding.

9. Professional Development Is Key for the Success of a Transition to Digital Technology and to a Conceptual Approach to Mathematics Instruction

The traditional belief has been that teachers with the strongest content background in mathematics produce better-educated students. But the National Mathematics Advisory Panel expressed concern that its task force had found no research that supported this premise. Shortly after publication of the panel's report, a study of middle school mathematics professional development was published in the *Journal for Research in Mathematics Education*. The study, involving 168 middle school mathematics teachers in Colorado, cited data that refute the idea that the amount of content knowledge or teaching longevity correlate positively to student achievement. The results indicated that a combination of deepening content knowledge coupled with pedagogical training focused on a problem-solving, inquiry-based approach had the most positive effect on raising student achievement.[42]

How Does Discovery Education Provide the Professional Development Needed to Help Teachers Make the Transition?

Discovery Education has extensive experience in providing professional development to hundreds of thousands of teachers that not only supports them in increasing their use of digital media, but also transitions their approach to instruction so that digital media is used to improve

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[41] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2013). *K–8 publishers' criteria for the Common Core State Standards for Mathematics.* Washington, DC: Authors. Retrieved from: http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Spring%202013_FINAL.pdf

[42] McMeeking, S. and Orsi, C. (2012). Effects of a teacher professional development program on the mathematics achievement of middle school students. *Journal of Research in Mathematics Education.43.2,* 159-181.

instruction and learning. It should be noted that Math Techbook is provided with professional development at no additional cost. Discovery Education enlists a partnership approach to professional development, customizing the professional development with the district and promoting a systemic change model. Such a model is founded on best practices in mathematics professional development as described in projects such as the National Science Foundation-funded teacher enhancement initiative.[43]

Discovery Education monitors the success of the professional development it delivers using assessments of teachers' digital integration skills based on the Technology Integration Matrix (TIM) developed by the Florida Center for Instructional Technology[44], which established behavioral benchmarks for teacher use of technology. The matrix defines five levels of use that correspond to both a transition to independence in using the technology as well as a transition to using the technology to facilitate higher-order learning activities. Those levels are Entry, Adoption, Adaptation, Infusion, and Transformation. Teachers progress from using the digital technology to deliver information to directing the use of digital resources by students, to guiding students to make independent choices about which digital resource is used and how it is used to solve a problem, and finally to a level where students use the digital media to share and collaborate on explanations and arguments.

Building on the TIM, Discovery Education developed the Technology and Digital Integration Survey (DIS) to assess teachers' use of technology and digital media in the classroom and their abilities to use technology in ways that are consistent with the stages of technology/media integration that have been identified in the research literature. Using a combination of the survey matched against data that indicates which types of resources are being used by teachers and students, the results of the studies are demonstrating that teachers who received strong professional development were most likely to move from the lowest to the highest levels of use.

Since 2011, the DIS has been implemented in more than 20 school districts with over 15,000 teachers. In our work with the Albuquerque Public Schools, during the first year of implementation of Discovery Education services, professional development activities were associated with improvement in the digital integration self-efficacy of 34 percent of teachers (Arroyo, 2012).[45] In addition, research showed that those teachers who were functioning at the highest levels of digital integration were more likely to use technology and to integrate digital media into their classroom lesson than those teachers who were functioning at the lowest levels on the research-based technology and digital integration continuum. Similar results have been found in our work with teachers in Canada school districts.

In Holy Trinity Catholic Schools District, where Discovery Education has implemented a three-year professional development initiative, the research has demonstrated that those teachers who completed Discovery Education professional development training were more likely than nonparticipants to integrate technology and digital media into their instruction. Furthermore, by the third year of the initiative, science teachers that completed Discovery Education training and had achieved the highest levels of digital integration, as assessed by the DIS, used more complex digital assets (e.g., interactives, virtual labs, simulations) that are known to inspire higher-order thinking among students, rather than to simply use videos or images within their lesson plans (Arroyo, 2013).[46]

In Rockford, Illinois, professional development that focused on using digital assessment results to

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[43] Banilower, E., Boyd, S, Pasley, J., Weiss, I. (December 2006). Lessons from a decade of mathematics and science reform: A capstone report for the local systemic change through teacher enhancement initiative. Prepared for the National Science Foundation by Horizon Research. Retrieved from: http://www.horizon-research.com/pdmathsci/htdocs/reports/capstone.pdf

[44] Allsopp, M. M., Hohlfeld, T., & Kemker, K. (2007). *The technology integration matrix: The development and field-test of an Internet based multi-media assessment tool for the implementation of instructional technology in the classroom.* Paper session presented at the annual meeting of the Florida Educational Research Association. Retrieved from http://fcit.usf.edu/matrix/resources.php

[45] Arroyo, C.G. (2012) Albuquerque public schools: Partnership evaluation results. Unpublished Data. Silver Spring, MD: Discovery Education. Available upon request.

[46] Arroyo, C.G. (2013). Holy Trinity Public Schools teacher survey. Unpublished Data. Silver Spring, MD: Discovery Education. Available upon request.

collaborate with colleagues and focus on instruction of individual students made significant differences in how educators in that district used assessments to inform, plan, and carry out instruction.[47]

Discovery Education Techbook implementation has proved equally impressive in terms of achievement. Significant improvement in state science scores for students in 87 Title I schools was recorded over two consecutive years in Charlotte-Mecklenburg Schools using DE Science digital resources.[48] Similar results have been reported for Collier County Public Schools in Florida for the use of Science Techbook[49] and in Indianapolis Public Schools in using social studies digital resources aligned to their curriculum.[50] Other studies with promising results are under way. It is clear that digital resources, coupled with exemplary professional development in their use, is a winning partnership.

Summary

The guiding principles that support Discovery Education's philosophy are based on solid research and best practices to improve mathematics teaching and learning. Based on extensive first-hand experience, Discovery Education believes that the combination of a carefully crafted Math Techbook as a complete primary resource for mathematics, and effective ongoing measured professional development, will produce significant gains in students' comprehension, proficiency, and appreciation of mathematics.

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[47] Altman, J, and Arroyo, C.G. (2013). Rockford County Public Schools teacher survey. Unpublished Data. Silver Spring, MD: Discovery Education. Available upon request.

[48] Arroyo, C.G. (2011) Narrowing the science achievement gap: The impact of Discovery Education Science on Title I students. Unpublished Data. Silver Spring, MD: Discovery Education. Available upon request.

[49] Arroyo, C.G. (2012) Use of Discovery Education Science Techbook in Colllier County Public Schools, academic year 2011-2012. Unpublished Data. Silver Spring, MD: Discovery Education. Available upon request.

[50] Arroyo, C.G. (2011) Accelerating social studies achievement: A study of the effectiveness of a Discovery Education Social Studies digital alignment strategy, Indianapolis Public Schools 2009-2011. Unpublished Data. Silver Spring, MD: Discovery Education. Available upon request.